CODE:2101- AG-19-23-24 पजियन क्रमांक **REG.NO:-TMC -D/79/89/36**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E

EXAMINATION 2023 -24

Time : 3 Hours — Maximum Marks : 80

CLASS – XII — **MATHEMATICS**

Sr. No.	SECTION – A **This section comprises of very short answer type-questions (VSA) of 2 marks each**	Marks
Q.1	Given, $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}, I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $A^2=5A+\lambda I$. Hence, λ = ? (a)23 (b)-23 (c) -7 (d) NONE	1
Q.2	Which of the following is not true (a) Every skew-symmetric matrix of odd order is non-singular (b) If determinant of a square matrix is non-zero, then it is non singular (c) Cofactor of symmetric matrix is symmetric(d)Cofactor of a diagonal matrix is diagonal	1
Q.3	If $A=\begin{bmatrix}\alpha & 2\\ 2 & \alpha\end{bmatrix}$ and $\lvert A^3\rvert$ =125, then α = (a) ± 3 (b) ± 2 (c) ± 5 (d) 0	1

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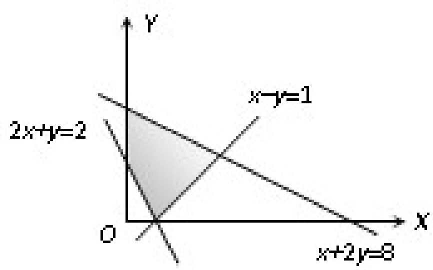
Q.4	If $f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2}, & \text{when } x<0\\ a, & \text{when } x=0\\ \dfrac{\sqrt{x}}{\sqrt{(16+\sqrt{x})}-4}, & \text{when } x>0\end{cases}$, is continuous at $x=0$, then the value of 'a' will be (a) 8 (b) -8 (c) 4 (d) None of these	1
Q.5	If θ be the angle between the unit vectors **a** and **b**, then $\mathbf{a}-\sqrt{2}\,\mathbf{b}$ will be a unit vector if $\theta =$ (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$	1
Q.6	Solution of $ydx - xdy = x^2 ydx$ is (a) $ye^{x^2} = cx^2$ (b) $ye^{-x^2} = cx^2$ (c) $y^2e^{x^2} = cx^2$ (d) $y^2e^{-x^2} = cx^2$	1
Q.7	For the following shaded area, the linear constraints except $x \geq 0$ and $y \geq 0$, are  (a) $2x+y \leq 2,\ x-y \leq 1,\ x+2y \leq 8$ (b) $2x+y \geq 2,\ x-y \leq 1,\ x+2y \leq 8$ (c) $2x+y \geq 2,\ x-y \geq 1,\ x+2y \leq 8$ (d) $2x+y \geq 2,\ x-y \geq 1,\ x+2y \geq 8$	1
Q.8	If vector $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ and vector $\mathbf{b} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, then $\dfrac{\text{Projection of vector } \mathbf{a} \text{ on vector } \mathbf{b}}{\text{Projection of vector } \mathbf{b} \text{ on vector } \mathbf{a}} =$ (a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) 3 (d) 7	1
Q.9	$\displaystyle\int_{-1}^{1} \frac{x^3+\lvert x\rvert+1}{x^2+2\lvert x\rvert+1}\,dx =$ (a) log 2 (b) 2log 2 (c) - log 2 (d) none of these	1

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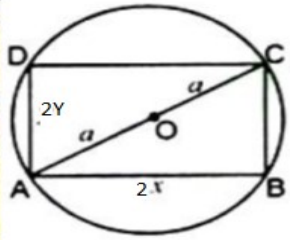
Q.10	If $A=\begin{vmatrix}-1 & 2 & 4\\ 3 & 1 & 0\\ -2 & 4 & 2\end{vmatrix}$ and $B=\begin{vmatrix}-2 & 4 & 2\\ 6 & 2 & 0\\ -2 & 4 & 8\end{vmatrix}$, then B is given by (a) $B=4A$ (b) $B=-4A$ (c) $B=-A$ (d) $B=6A$	1
Q.11	Two tailors A and B earn Rs. 15 and Rs. 20 per day respectively A can make 6 shirts and 4 pants in a day while B can make10 shirts and 3 pants. To spend minimum on 60 shirts and 40 pants, A and B work x and y days respectively. Then linear constraints except $x \geq 0$, $y \geq 0$, are (a) $15x + 20y \geq 0, 60x + 40y \geq 0$ (b) $15x + 20y \geq 0, 6x + 10y = 10$ (c) $6x + 10y \geq 60, 4x + 3y \geq 40$ (d) $6x + 10y \leq 60, 4x + 3y \leq 40$	1
Q.12	If $\lvert \mathbf{a} \rvert = \lvert \mathbf{b} \rvert = 1$ and $\lvert \mathbf{a}+\mathbf{b} \rvert = \sqrt{3}$, then the value of $(3\mathbf{a}-4\mathbf{b}).(2\mathbf{a}+5\mathbf{b})$ is (a) – 21 (b) –21/2(c) 21 (d) 21/2	1
Q.13	If I is a unit matrix of order 10, then the determinant of I is equal to (a) 10 (b) 1 (c) 1/10 (d) 9	1
Q.14	Three coins are tossed. If one of them shows tail, then the probability that all three coins show tail, is (a) $\frac{1}{7}$ (b) $\frac{1}{8}$ (c) $\frac{2}{7}$ (d) $\frac{1}{6}$	1
Q.15	Integrating factor of $\frac{dy}{dx}+\frac{y}{x}=x^3-3$ is (a) x (b) $\log x$ (c) $-x$ (d) e^x	1
Q.16	The function $f: R \to R, f(x)=x^2, \forall x \in R$ is (a) Injection but not surjection (b)Surjection but not injection (c) Injection as well as surjection (d) Neither injection nor surjection	1
Q.17	If $f(x)=\begin{cases} x+\lambda, & x<3\\ 4, & x=3\\ 3x-5, & x>3\end{cases}$ is continuous at $x = 3$, then $\lambda =$ (a) 4 (b) 3(c) 2 (d) 1	1
Q.18	If a line makes angles of 30^o and 45^o with x-axis and y-axis, then the angle made by it with $z-$axis is (a) 45^o (b) 60^o (c) 120^o (d) None of these	1
	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is	

	true.	
Q.19	**Assertion (A) :** The point of the function $(x-1)(x-2)^2$ at its maxima is $\frac{4}{3}$. **Reason (R) :** $f'(c)$ changes sign from positive to negative as x increases through c then the function attains a local maximum at x = c .	1
Q.20	Assertion (A): If $\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}\bullet\vec{b}\right)^2=225\ \&\ \left\|\vec{a}\right\|=5$ then the value of $\left\|\vec{b}\right\|=3$ Reason (R): $\left\|\vec{a}\times\vec{b}\right\|^2+(\vec{a}.\vec{b})^2=\left\|\vec{a}\right\|^2\left\|\vec{b}\right\|^2$	1
	SECTION – B **This section comprises of very short answer type-questions (VSA) of 2 marks each**	
Q.21	Find the intervals in which the function f given by $f(x)=x^3+\frac{1}{x^3}, x\neq 0$ is (i) increasing (ii) decreasing .	2
Q.22	Prove that : $\tan^{-1}\left[\frac{\left\{\sqrt{1+x^2}+\sqrt{1-x^2}\right\}}{\left\{\sqrt{1+x^2}-\sqrt{1-x^2}\right\}}\right]=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$. **OR** Write $\tan^{-1}\left[\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right],\ x\in\left(\pi,\frac{3\pi}{2}\right)$ in the simplest form .	2
Q.23	For any vectors $\vec{a}$, show that $\left\|\vec{a}\times i\right\|^2+\left\|\vec{a}\times j\right\|^2+\left\|\vec{a}\times k\right\|^2=2\left\|\vec{a}\right\|^2$	2
Q.24	Find the maximum slope of the curve $y=-x^3+3x^2+2x-27$. **OR** Separate the interval $[0,\frac{\pi}{2}]$ into sub intervals in which $f(x)=\sin^4 x+\cos^4 x$ is increasing or decreasing .	2
Q.25	A man 2 metres high walks at a uniform speed of 5 km/hr away from a lamp - post 6 metres high. Find the rate at which the length of his shadow increases.	2
	SECTION – C **(This section comprises of short answer type questions (SA) of 3 marks each)**	
Q.26	Evaluate : $\int\frac{1}{\sin x-\sin 2x}dx$.	3
Q.27	Evaluate: $\int\frac{x^2}{x^4+x^2+16}dx$. **OR**	3

	Evaluate : $\int_{-1}^{\frac{1}{2}} \lvert x \cos(\pi x) \rvert dx$.	
Q.28	The ratio of the number of boys to the number of girls in a class is 1 : 2 . It is known that the probabilities of a girl and boy getting a first division are .25 and .28 respectively . Find the probability that a student chosen at random will get first division. **OR** From a set of 100 cards numbered 1 to 100, one card is drawn at random. Find the probability that the number on the card is divisible by 6 or 8, but not by 24.	3
Q.29	Solve the differential equation $\left(x^2 - yx^2\right)dy + (x^2y^2 + y^2)dx = 0$ given that y = 1 when x = 1 . **OR** Prove that $x^2 - y^2 = c(x^2 + y^2)^2$ is the general solution of the differential equation $\left(x^3 - 3xy^2\right)dx = c(y^3 - 3x^2y)dy$ where c is a parameter .	3
Q.30	If $x\sqrt{(1+y)} + y\sqrt{(1+x)} = 0$ then $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.	3
Q.31	Solve the following linear programming problem (L.P.P) graphically. Maximize $Z = x + 2y$ subject to constraints ; $x + 2y \geq 100$ $2x - y \leq 0$ $2x + y \leq 200$ $x, y \geq 0$.	3
	SECTION – D **(This section comprises of long answer-type questions (LA) of 5 marks each)**	
Q.32	Determine the equation of the line passing through the point (1, 2, -4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. **OR** Find the equations of the line which intersects the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and passes through the point (1, 1, 1) .	5
Q.33	Using integration, find the area of the triangle bounded by the lines 11 = 7x - 2y , 19 = 3x + 2y and x – y = 3.	5
Q.34	Check whether the relation R on R defined as R = $\{(a, b)$: $a \leq b^3\}$ is reflexive , symmetric or transitive. **OR** Prove that the function $f : [0, \infty) \to R$ Given by f (x) = 9x^2 + 6x – 5 is not invertible. Modify the co-domain of the function f to make it invertible, and hence find f^{-1} .	5
Q.35	If A and B are two independent events such that $P(A \cap B) = \frac{1}{6}$ and $P\left(\bar{A} \cap \bar{B}\right) = \frac{1}{3}$, find P (A) &P (B) .	5

	SECTION – E (This section comprises of 3 case study / passage – based questions of 4 marks each with two sub parts (i),(ii),(iii) of marks 1, 1, 2 respectively.The third case study question has two sub – parts of 2 marks each.)	
Q.36	**Case Study based-1** If there is a statement involving the natural number n such that (i) The statement is true for $n=1$ (ii) When the statement is true for $n=k$ (where k is some positive integer), then the statement is also true for $n=k+1$. Then, the statement is true for all natural numbers n. Also, if A is a square matrix of order n, then A^2 is defined as AA. In general, $A^m=AA..A$(m times), where m is any positive integer. Based on the above information, answer the following questions.	
i.	If $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, then for any positive integer n, (a) $A^n=\begin{bmatrix}3n & -4n\\ n & -n\end{bmatrix}$ (b) $A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$ (c) $A^n=\begin{bmatrix}3n & -8n\\ 1 & -n\end{bmatrix}$ (d) $A^n=\begin{bmatrix}1+3n & -4n\\ n & 1-3n\end{bmatrix}$	1
ii.	If $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$, then $\lvert A^n\rvert$, where $n\in N$, is equal to (a) 2^n (b) 3^n (c) n (d) 1	1
iii.	If $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then which of the following holds for all natural numbers $n\geq 1$? (a) $A^n=nA-(n-1)I$ (b) $A^n=2^{n-1}A-(n-1)I$ (c) $A^n=nA+(n-1)I$ (d) $A^n=2^{n-1}A+(n-1)I$ **OR** Let $A=\begin{bmatrix}a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & a\end{bmatrix}$ and $A^n=[a_{ij}]_{3\times 3}$ for some positive integer n, then the cofactor of a_{13} is (a) a^n (b) $-a^n$ (c) $2a^n$ (d) 0	2
Q.37	**Case Study based-3** A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)	

	 	
i.	The perimeter of rectangle P is: a. $4x+4\sqrt{a^2-x^2}$ b. $x+\sqrt{a^2-x^2}$ c. $4x+\sqrt{a^2-x^2}$ d. $x+4\sqrt{a^2-x^2}$	1
ii.	To find the critical points put a. $\frac{dP}{dx}>0$ b. $\frac{dP}{dx}<0$ c. $\frac{dP}{dx}=0$ d. None of these	1
iii.	Value of y is a. $\frac{a}{2}$ b. $\frac{a}{\sqrt{2}}$ c. 2a d. $\sqrt{2}a$ OR If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square then the sides of the region a. $10\sqrt{8}$ cm b. $2\sqrt{10}$ cm c. $20\sqrt{2}$ cm d. $10\sqrt{2}$ cm	2
Q.38	**Case Study based-3** From the point (2, 4, -1) to the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$.	
i.	Find the equation of the perpendicular from the point on the line the length of perpendicular.	2
ii.	The length of perpendicular.	2
	“अवसर की प्रतीक्षा में मत बैठो । आज का अवसर ही सर्वोत्तम है ।”	